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## Antidot lattice in QHE regime: macroscopic limit

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**Abstract.** QHE is studied in 2D system with antidots. The size of antidots is considered large compared with quantum and relaxation lengths. In this limit the electric field in the system is described by a continuity equation. It is found, that the electric field in the system without conducting boundaries can be expressed through that in the same system without magnetic field. The electric field and the current density are found in structures, containing one and two antidots and in a round disk with point contacts as well.

### Introduction

The different descriptions of quantum Hall effect (QHE) results in the same quantization. They are microscopic approach, based on local relation between the electric field and the current density, the edge current approach, attributing the Hall quantization to the boundary currents, and the macroscopic approach.

Unlike the standard theories of QHE, based on microscopic consideration of conductivity tensor in a homogeneous electric field not taking into account the spatial fluctuations of external field, some recent works consider the Hall quantization as a macroscopic phenomenon, using the quantized Hall conductivity and zero drift conductivity of an ideal system as a zero approximation, and solving the problem of current flow in a mixture of Hall conductors and normal metal [1] or Hall conductors on different plateaus [2, 3]. This approach gave so called "semicircle" relation between the Hall and drift components, not containing the Plank constant, correctly describing the experimental data both for integer and fractional Hall effect.

The purpose of the present paper is the study of electric field and current distribution of nonhomogeneous quantum Hall system. We shall consider the quantum Hall system as a mixture of an homogeneous Hall conductor and insulating phase, neglecting the screening length. Really, such system represents, for example, the QH system with strong doping and compensation, where the insulating domains are formed by randomly reduced density of donors. In particular, in the extreme quantum limit (one partly filled Landau level) the system can be considered as a mixture of insulating domains, where the first Landau level is empty and QHE insulator domains, where the first Landau level is filled. The other example provides the hand-made potential relief of so called antidot lattice, where the problem of current flow has independent meaning, in particular for the QHE breakdown [4].

### 1 Reduction of current and electric field distribution to zero-magnetic field problem.

We consider the 2D Hall conductor with common insulator inclusions. We based on the local expression for the density of current  $\mathbf{j} = \hat{\sigma} \nabla \phi$  obeying the continuity equation  $\nabla \mathbf{j} = 0$  with boundary conditions  $(\mathbf{j}\mathbf{n}) = 0$ , where  $\hat{\sigma} = ||\sigma_{ij}||$  is the conductivity tensor,  $\phi$  is potential and  $\mathbf{n}$  is a normal to the insulator boundary. The continuity equation gives the Laplace equation for potential  $\sigma_{xx} \Delta \phi = 0$ .

Unlike the usual formulation of conditions of problem of field distribution, *including* extended conducting contacts, the problem with insulating boundaries (and may be point current contacts) has a very simple solution. It can be found by mapping the problem with magnetic field to the same one without magnetic field, solved by means of the theory of analytical functions.

Consider the holomorphic function  $w = \phi_0 + i\psi_0$ , where  $\phi_0$  and  $\psi_0$  satisfy the boundary conditions

$$\mathbf{n} \times \nabla \phi_0 = 0 \quad \mathbf{n} \nabla \psi_0 = 0, \quad \int \nabla \psi_0 dS/S = \mathbf{E}_0, \quad (1)$$

where  $S$  is the sample area, the mean electric field  $\mathbf{E}_0$  is applied along  $x$  axes.

The potential  $\psi_0$  supplied by expression for current  $\mathbf{j} = \sigma \nabla \psi_0$ , corresponds to the problem of current flow in a medium with local longitudinal conductivity  $\sigma$  with insulator inclusions in the absence of magnetic field. The potential  $\phi_0$  corresponds to current flow in conducting medium with ideally-conducting inclusion of the same form. This potential is also the solution of current flow problem in ideal QHE system ( $\sigma_{xx} = 0$ ) with insulating inclusion.

Let the function  $\phi$  is defined by an equation

$$\phi = \sin \alpha \phi_0 + \cos \alpha \psi_0 = \text{Im}(e^{i\alpha} w). \quad (2)$$

The function  $\phi$  satisfies the condition  $\mathbf{n} \hat{\sigma} \nabla \phi = 0$  if the angle  $\alpha$  is

$$\cos \alpha = \frac{\sigma_{xx}}{\sqrt{\sigma_{xx}^2 + \sigma_{xy}^2}}, \quad 0 < \alpha < \pi/2. \quad (3)$$

As a result, the solution of the boundary problem  $\phi$  with magnetic field  $B$  differs from that without magnetic field  $\psi_0$  by the rotation of the vector of electric field in any point on the angle  $\alpha$ :  $\mathbf{E}(\mathbf{r}, B) = \hat{U}(\alpha)\mathbf{E}(\mathbf{r}, 0)$ , where  $\hat{U}(\alpha)$  is a matrix of rotation on angle  $\alpha$ .

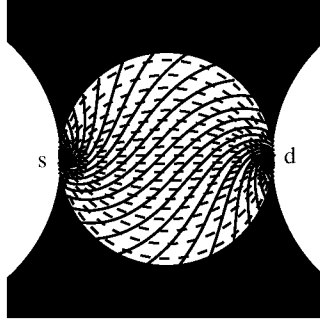
## 2 Electric field and current distribution in confined systems

Three problems of field and current distribution were solved analytically. One is the density of current in a round quantum disk with point tunnel contacts. Another are current and field distribution around one and two antidots. The figures show the equipotential lines and lines of current in these cases. As seen from figures, the magnetic field does not change the current distribution for the fixed current on the external boundary and leads to the rotation of electric field, while the fixed external field results in the rotation of the whole picture of current together with the current on the boundary.

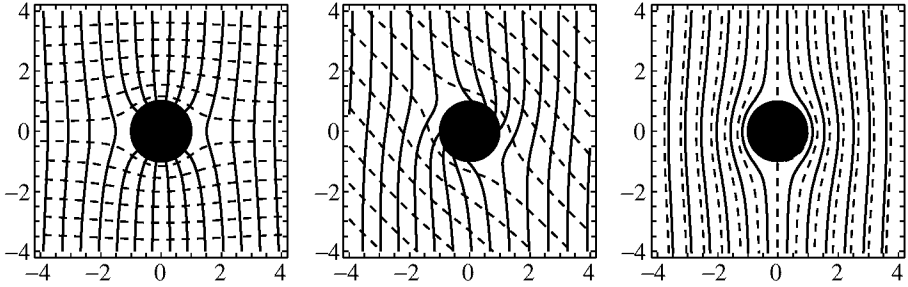
## 3 Discussion

Besides the considered cases of field and current distribution, the analytical solutions of field and current distribution problems can be found for domains with known conformal map. For example, it can be done for insulator inclusion with fractal form [10].

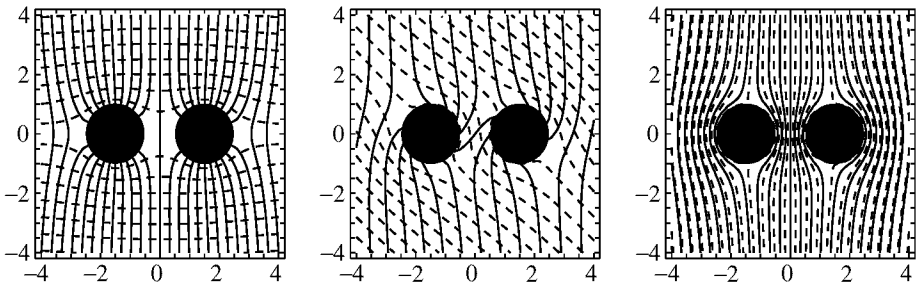
Another problem, which is tightly bound with considered here, is the problem of effective conductivity of antidot system in QHE regime [11]. We expressed the effective Hall and drift conductivities of Hall conductor through the effective conductivity of geometrically equivalent system in zero magnetic field. It is important to emphasize that in the QHE plateau regime when the local drift conductivity goes to zero, the effective drift and Hall



**Fig. 1.** Equipotential lines (solid) and current lines (dashed) in a round large quantum dot with two tunnel contacts, according to analytic formulae  $j_x - ij_y = 2aJ[\pi(a^2 - z^2)]^{-1}$ ,  $\phi = J\pi^{-1}(\sigma_{xx}^2 + \sigma_{xy}^2)^{-1/2}\text{Re}[\exp(-i\alpha)\log(a - z) - \log(a + z)]$ .  $J$  is the overall current,  $z = x + iy$ . The angle  $\alpha$  is  $\pi/4$ .



**Fig. 2.** Current and field distribution around one antidot (black), determined by formula  $E^*(z) = E^*(\infty) - E(\infty)e^{-2i\alpha}a^2/z^2$ , where  $E(z) = E_x + iE_y$ . The mean electric field has  $x$ -direction. The angle  $\alpha$  runs from left to right values  $0$  ( $B = 0$ ),  $\pi/4$  and  $\pi/2$  (Hall plateau).



**Fig. 3.** The same as Fig. 2 for two antidots. The field distribution is found analogically to [5].

conductivities coincide with local ones. We also used the solution of current and field distribution problem for description of QHE breakdown in antidot arrays.

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